Extended MHD Analysis of the Gravitational Interchange.

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In a recent letter, Zhu, et al. derive a dispersion relation for the g-mode in a slab using an extended MHD model that includes gyroviscosity and a two-fluid Ohm's law [P. Zhu et al, PRL 101, 2008]. The authors analyze the dispersion relation in the simplified case that only includes gyroviscosity. They show that complete gyroviscous stabilization can fail with finite-β. Here we analyze the dispersion relation that results from including two-fluid effects but neglecting gyroviscosity, and we analyze the dispersion relation that results from including both effects. We also present calculations of the g-mode comparing results from the NIMROD code [Sovinec and King, JCP 229, 2010] with the analytic model.

The two-fluid dispersion relation reduces to a two-parameter model

$$X^3 + G\,(1 + H)\,X^2 + X + GH = 0,$$

where $X = \omega/\gamma_M$ is the mode frequency normalized to the MHD growth rate, $\gamma_M = gn'/n$, $G = \omega_g/\gamma_M$ is the normalized gravitational drift frequency, $\omega_g = -gk_\perp/\Omega$, and $H = \omega_*/\omega_g$ is a normalized ion diamagnetic drift frequency, $\omega_* = \frac{k_\perp}{\Omega}\frac{1}{m_i n}\left(P_i' - \gamma P_i \frac{n'}{n}\right)$. The behavior of the solutions to the dispersion relation is determined by H, which only depends on equilibrium quantities. Increasing G corresponds to increasing $k_\perp$. At small G the solutions are the unstable g-mode, its damped complex conjugate counterpart, and a stable ion drift wave. For $H > 0$ a second instability exists due to the interaction between the g-mode and the ion drift wave. This instability grows at a rate comparable to the MHD growth rate and persists at infinite $k_\perp$.

A second instability also exists in the full model. Its growth rate greatly exceeds the MHD growth rate and results from the interaction between the two branches of the stabilized g-mode. The instability only occurs when $k_\perp \rho_i > 1$, where ρ_i is the ion Larmor radius. While the fluid model is not physically valid in this regime, this mode is a concern for codes that use extended MHD.

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