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Huygens' principle-based wavefront tracing in non-uniform media

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Abstract

We present preliminary results on a novel numerical method describing wave propagation in non-uniform media. The method could be applied to electromagnetic and electrostatic waves in non-magnetized plasmas. We also outline how to extend it to anisotropic media such as magnetized plasmas. The method is inspired by Huygens' principle, in the sense that it models the wavefront as an array of point sources. The spatial density of points and the power distribution among them are established according to Gauss-Hermite quadrature for Gaussian beams, or adapted quadrature in more general settings. These point sources emit wavelets, which interfere. In principle the electric field could be evaluated in a region in front of the original wavefront, and a new iso-phase surface could be identified. However, more simply and more quickly, a bisection method is used here to localize the nearest zeroes of electric field. This corresponds to a specific phase-increment from one wavefront to the other, but it is easily generalized by anticipating or delaying all the initial phases by the same amount. A simple reiteration of this method allows to trace wavefronts and their intensity profiles. The method is more time-consuming than ray tracings, but it accounts for diffraction. Two examples provided are diffraction around an obstacle and the finite waist of a focused Gaussian beam. The calculations were performed in two dimensions, but can be easily extended to three dimensions. We will also discuss the extension to anisotropic media by means of anisotropically expanding (i.e., ellipsoidal) wavelets.

Motivation and potential advantages

- Fast and accurate numerical method for wave propagation in non-uniform media.
- Applicable to e.m. and e.s. waves in non-magnetized plasmas.
- Can be extended by means of ‘ellipsoidal wavelets’ to anisotropic media such as magnetized plasmas.

- More time-consuming than ray tracings, but it accounts for diffraction.
- Two examples provided here:
  - diffraction around an obstacle
  - finite waist of a focused Gaussian beam
- Future work will include benchmark against full-wave methods, and assessment of pros and cons.

Numerical method

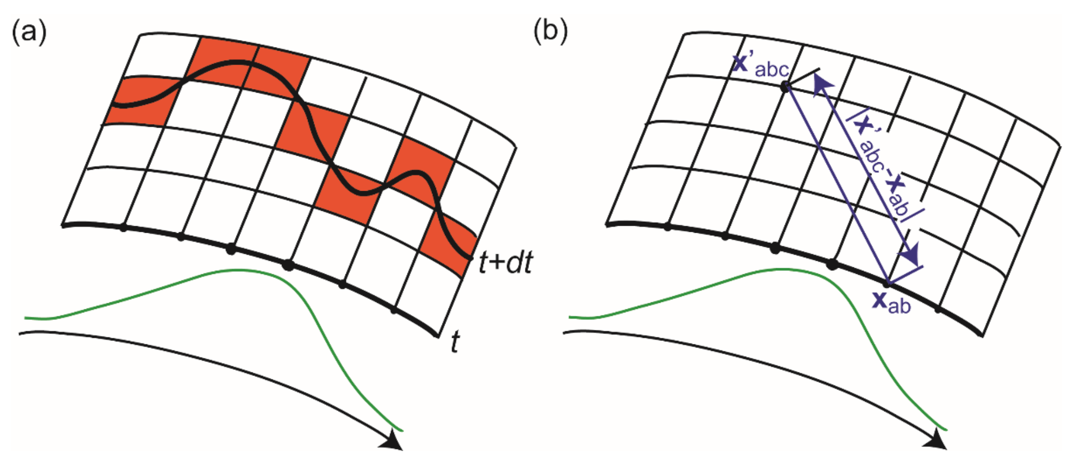
- Wavefront as an array of point sources.
  - Density of points and power distribution according to: (1) uniform distribution, (2) Gauss-Hermite quadrature for Gaussian beams, (3) adapted quadrature.
- Point sources emit wavelets, which interfere.
- Next wavefront calculated by searching for locus of (easy) iso-phase → zeroes of E.
  - Generalized by phase-anticipating or delaying the sources.
- Two possible approaches:
  - Maxwell solver in a region in front of the original wavefront.

  → Zero search , e.g. by bisection (simpler, quicker)

- Method is reiterated →

  → wavefronts &

  intensity profiles.



Interference calculations

In 2D

$$d\mathbf{E}(\mathbf{x}') = \mathbf{A}(\mathbf{x}) \frac{e^{ik|\mathbf{x}'-\mathbf{x}|}}{\sqrt{|\mathbf{x}'-\mathbf{x}|}} dl$$

In 3D

$$d\mathbf{E}(\mathbf{x}') = \mathbf{A}(\mathbf{x}) \frac{e^{ik|\mathbf{x}'-\mathbf{x}|}}{|\mathbf{x}'-\mathbf{x}|} dS$$

Discretizing and summing over all sources $_{ab}$ gives component E_i in target point $_c$:

$$E_{i,abc} = \sum_{a=1}^{m} \sum_{b=1}^{n} A_{i,ab} \frac{\exp ik_O F_{abc}}{F_{abc}}$$

where

$$F_{abc} = |\mathbf{x}'_{i,abc} - \mathbf{x}_{i,ab}|$$

or, in anisotropic media,

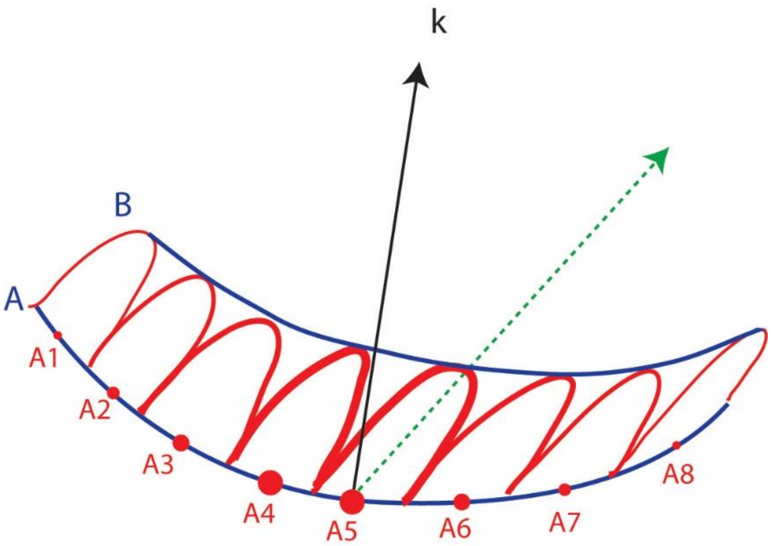
$$F_{abc} = \sqrt{\frac{\left(x'_{i,abc} - x_{i,ab}\right)^T M_{ij}^{-1} \left(x'_{j,abc} - x_{j,ab}\right)}{|M_{ij}^{-1}|}}$$

'Ellipsoidal wavelets' can extend Huygens' principle to anisotropic media

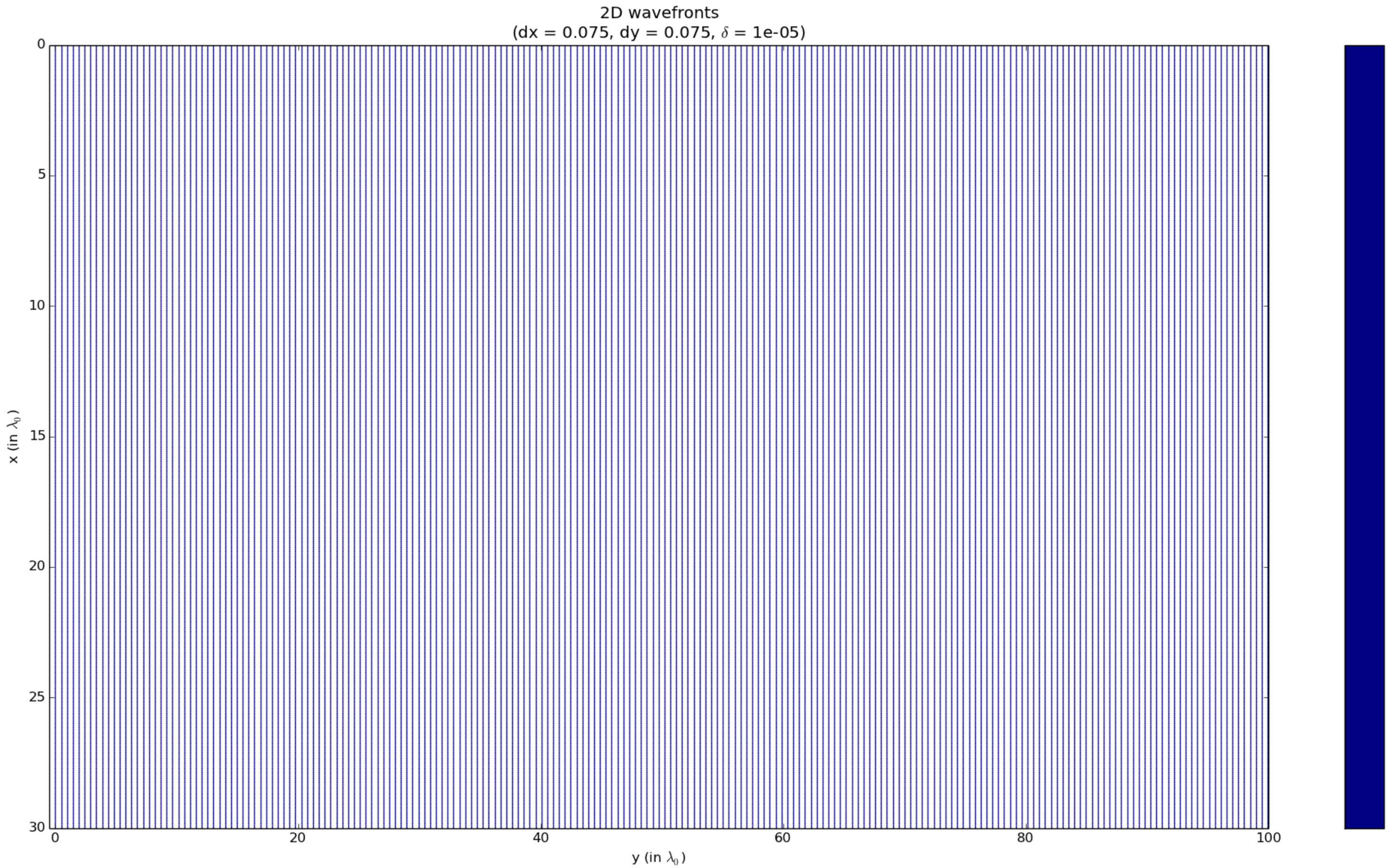
$$|\mathbf{x}' - \mathbf{x}| \rightarrow \sqrt{\frac{(\mathbf{x}' - \mathbf{x})^T M^{-1} (\mathbf{x}' - \mathbf{x})}{|M^{-1}|}}$$

where *M* defines the semi-axes of the ellipsoid.
M is polarization-dependent (O, X, B).

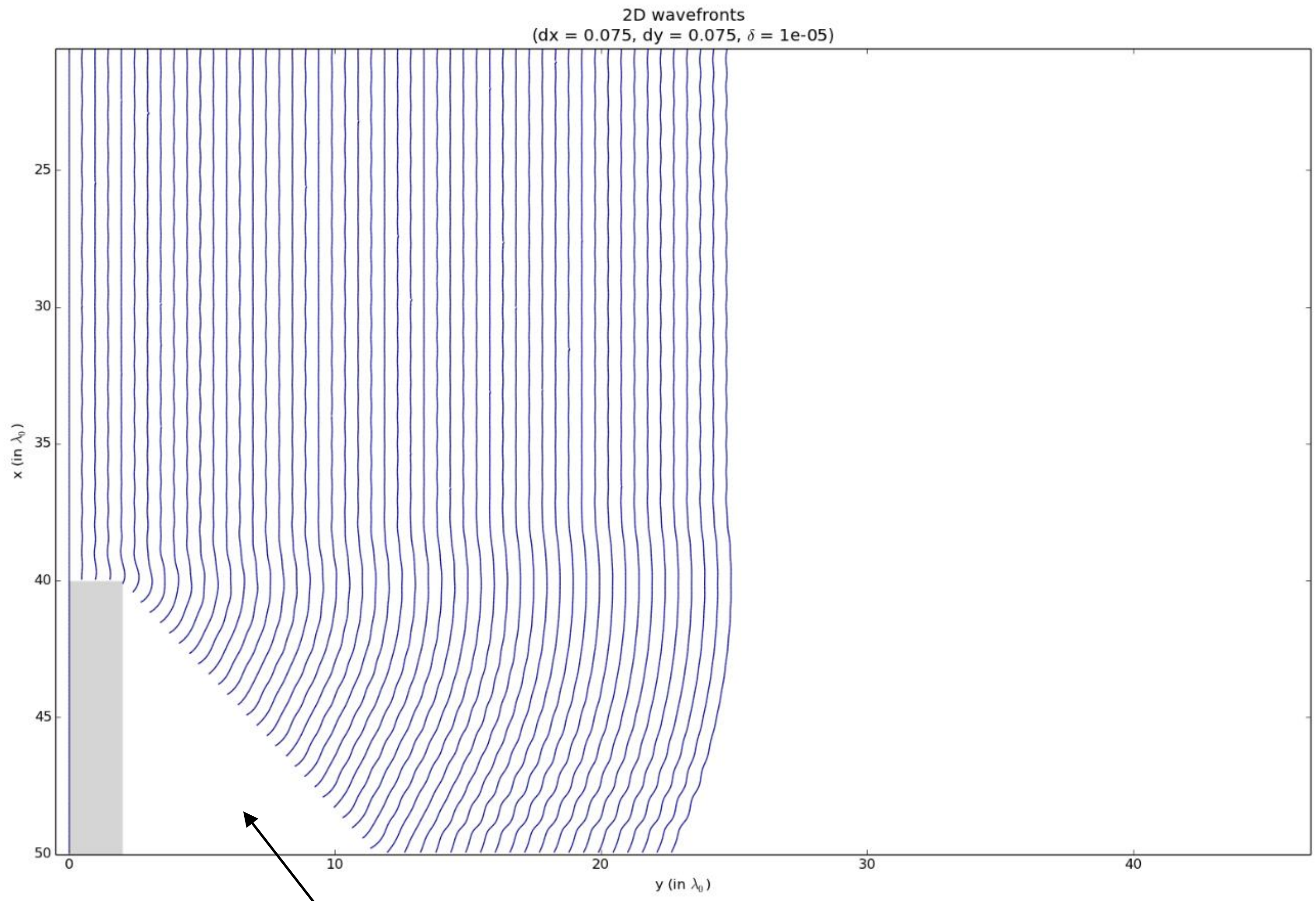
$$\mathbf{E}_O(\mathbf{x}') = \oiint \mathbf{A}_O(\mathbf{x}) \frac{\exp\left\{ik_O\sqrt{\frac{(\mathbf{x}' - \mathbf{x})^T {M_O}^{-1} (\mathbf{x}' - \mathbf{x})}{|{M_O}^{-1}|}}\right\}}{\sqrt{\frac{(\mathbf{x}' - \mathbf{x})^T {M_O}^{-1} (\mathbf{x}' - \mathbf{x})}{|{M_O}^{-1}|}}} dS$$



Planar waves in uniform medium



Diffraction around an obstacle



Polar grid, rather than Cartesian,
to be used for zero search in this region.

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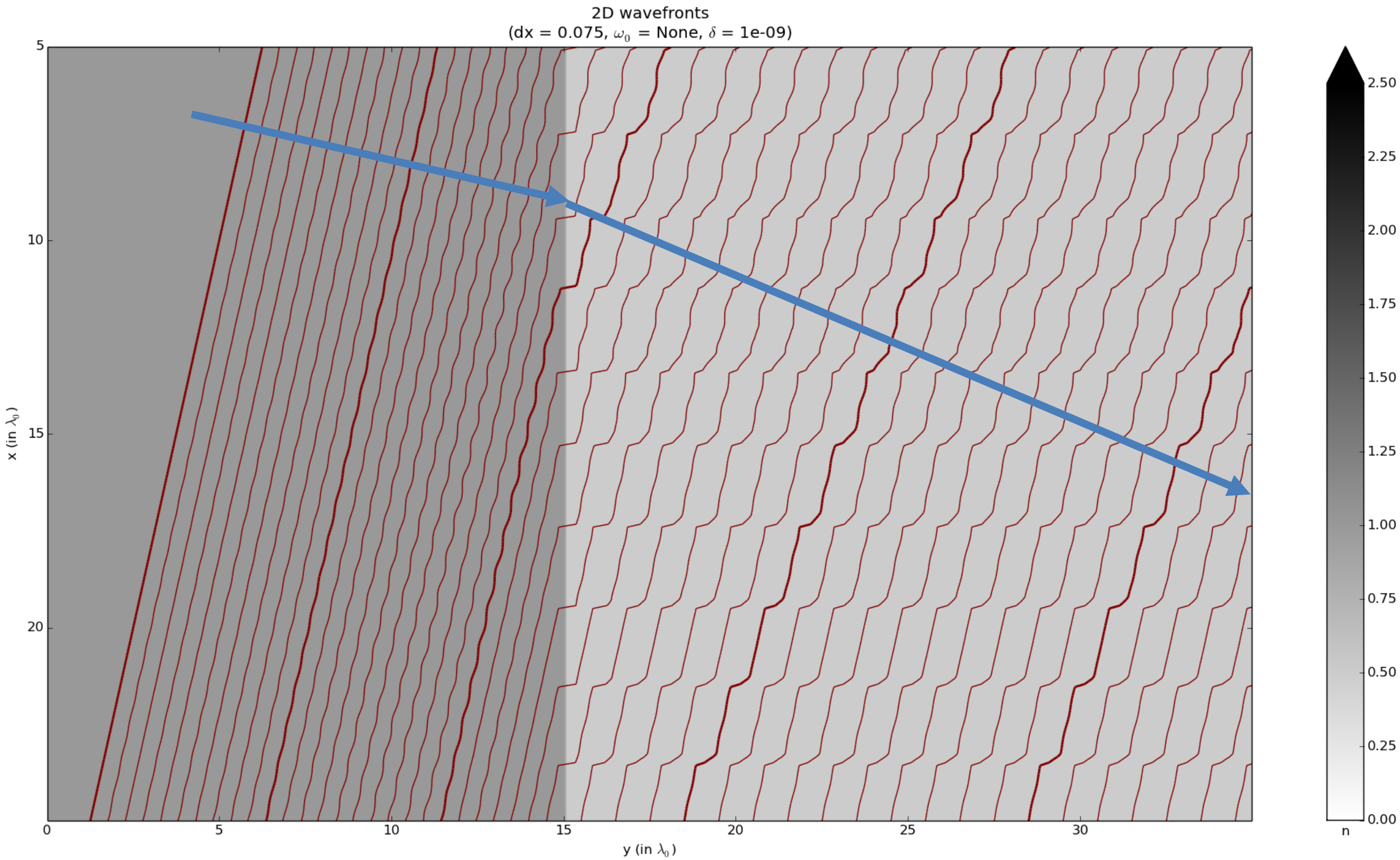
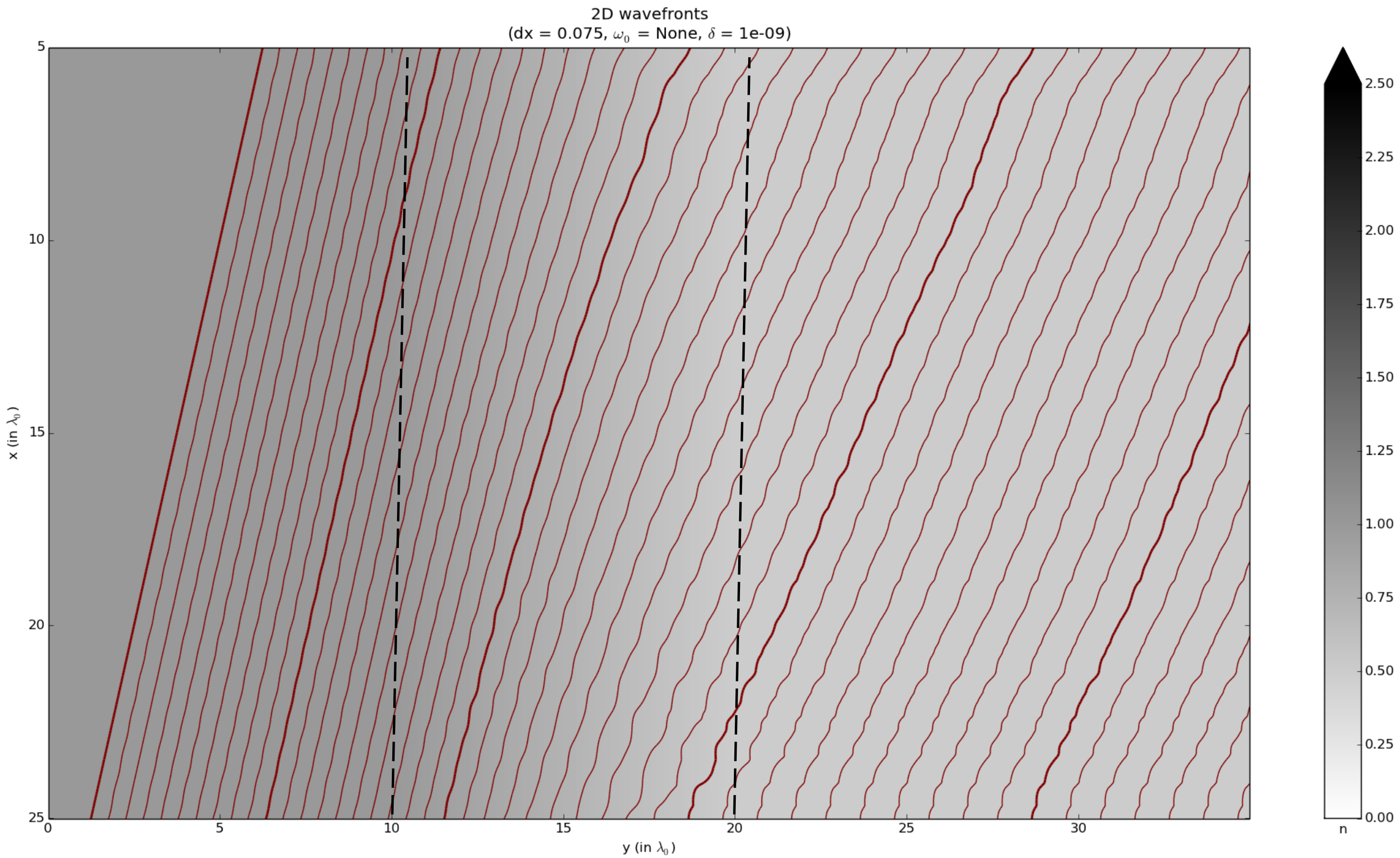
Snell's law

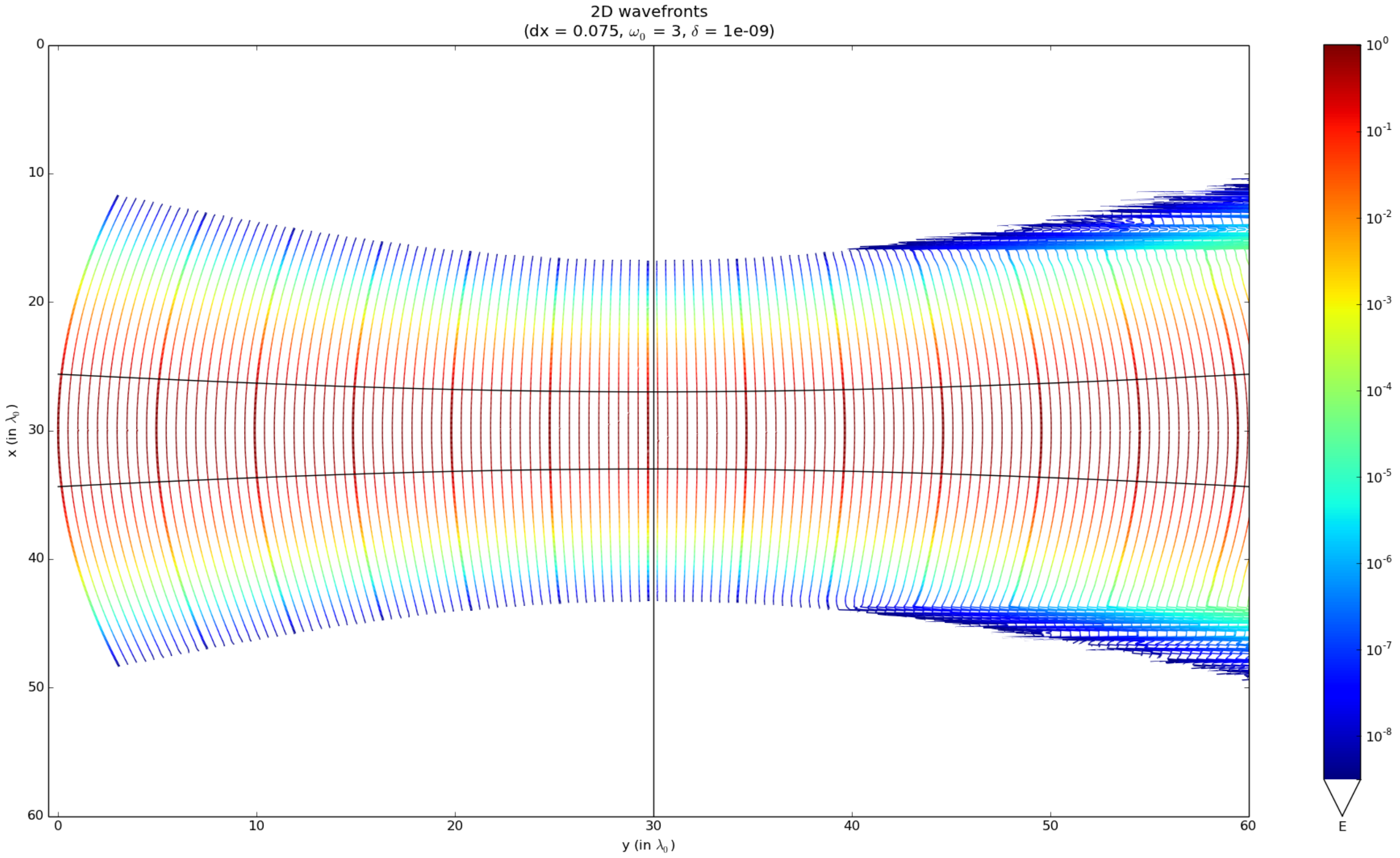


Corrugations, due to lack of ghost cells, are amplified by discontinuity. *Ongoing work*: add ghost cells and split source → target propagation into source → discontinuity → target (with different **k**).

Refraction in slowly varying medium



Gaussian beam, electric field



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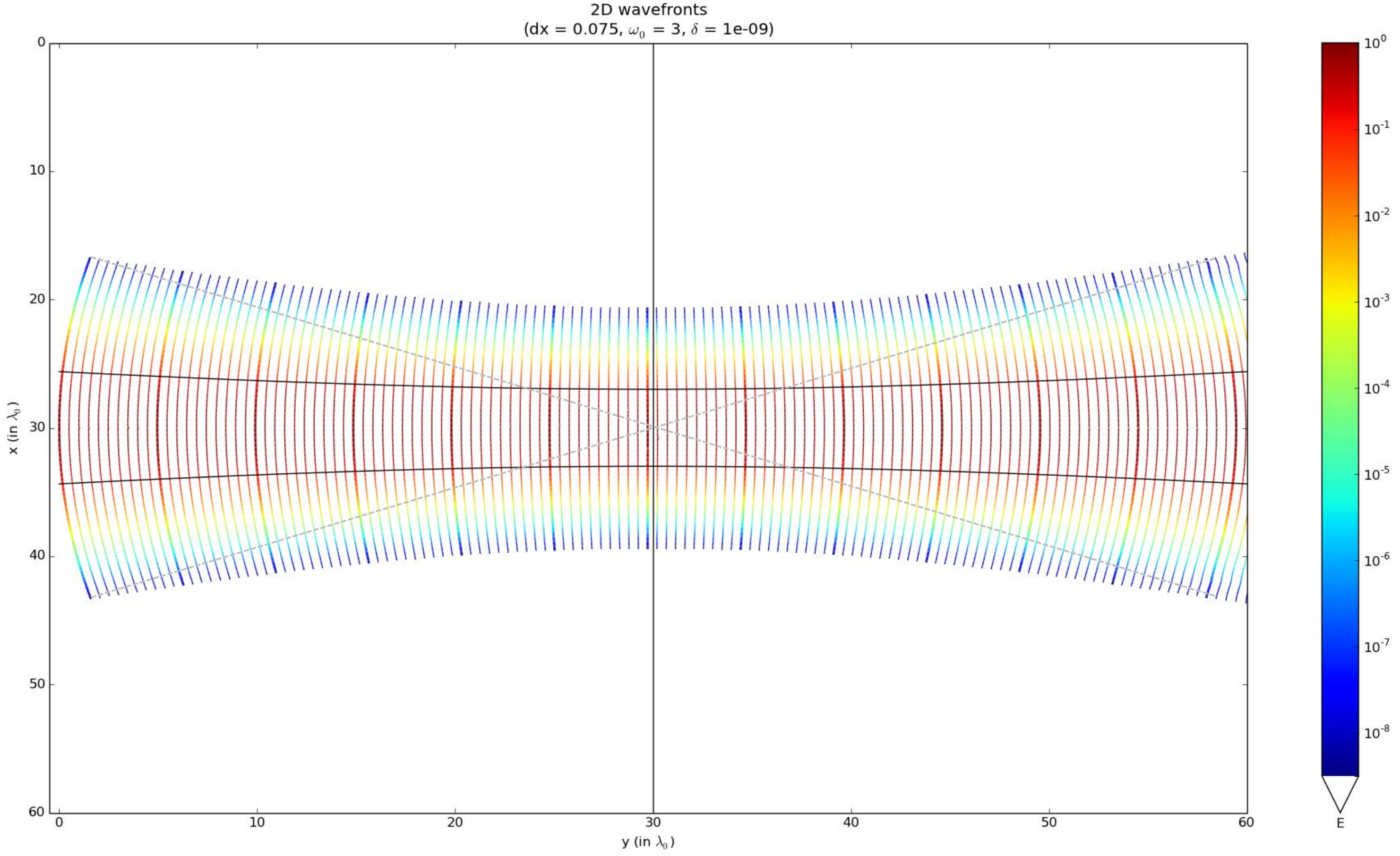
Gaussian beam, intensity



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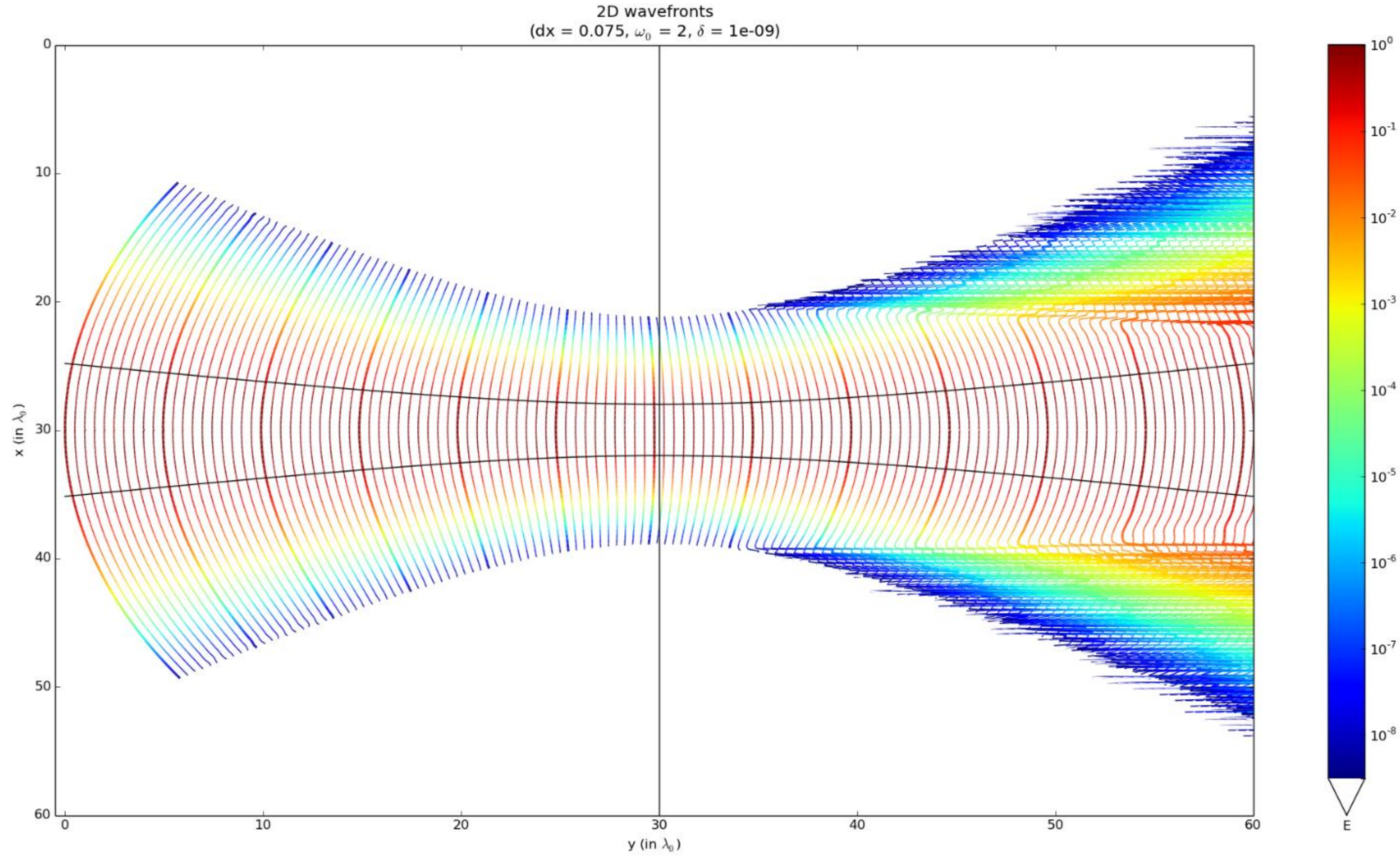
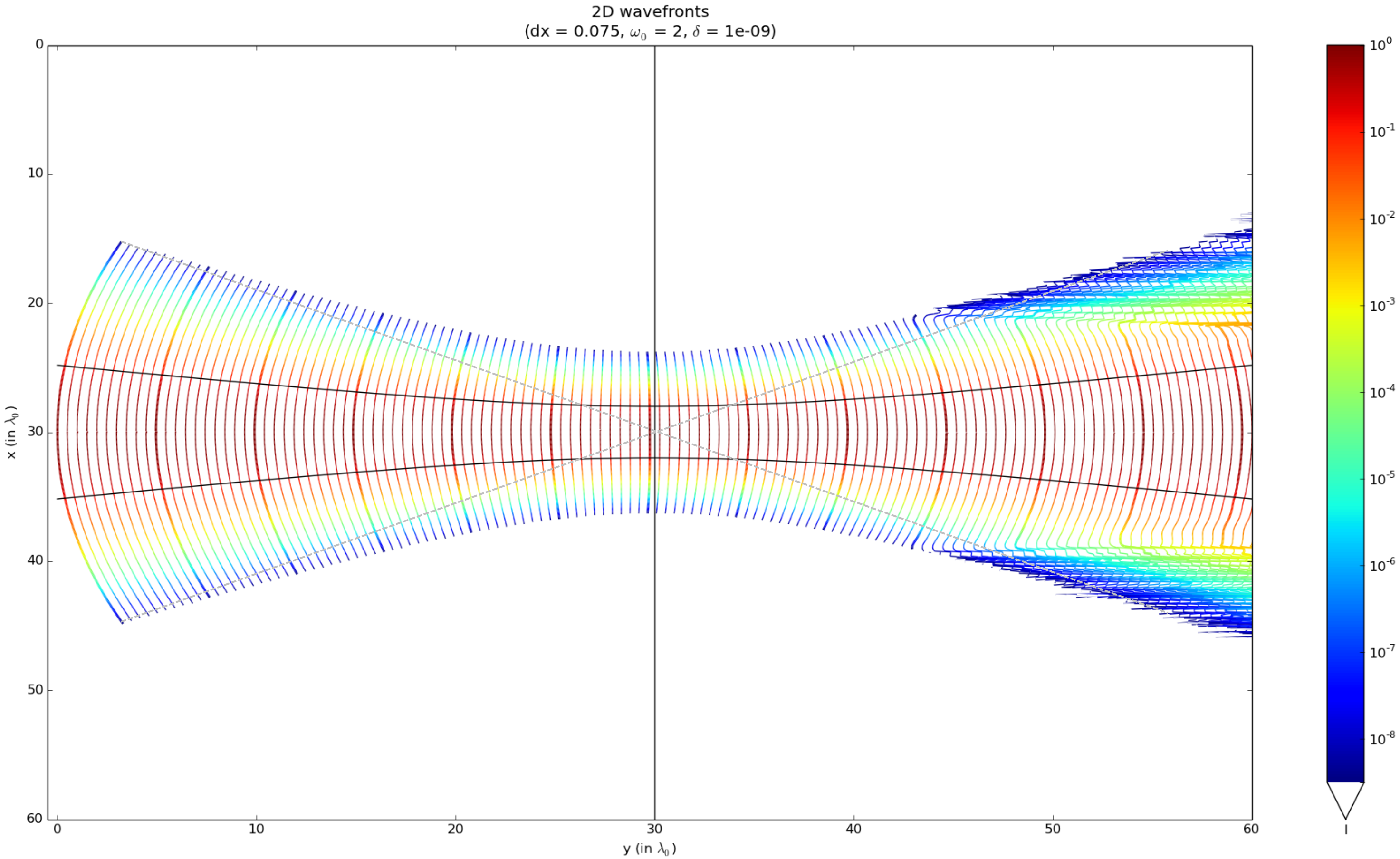
Strongly focused Gaussian beam, E

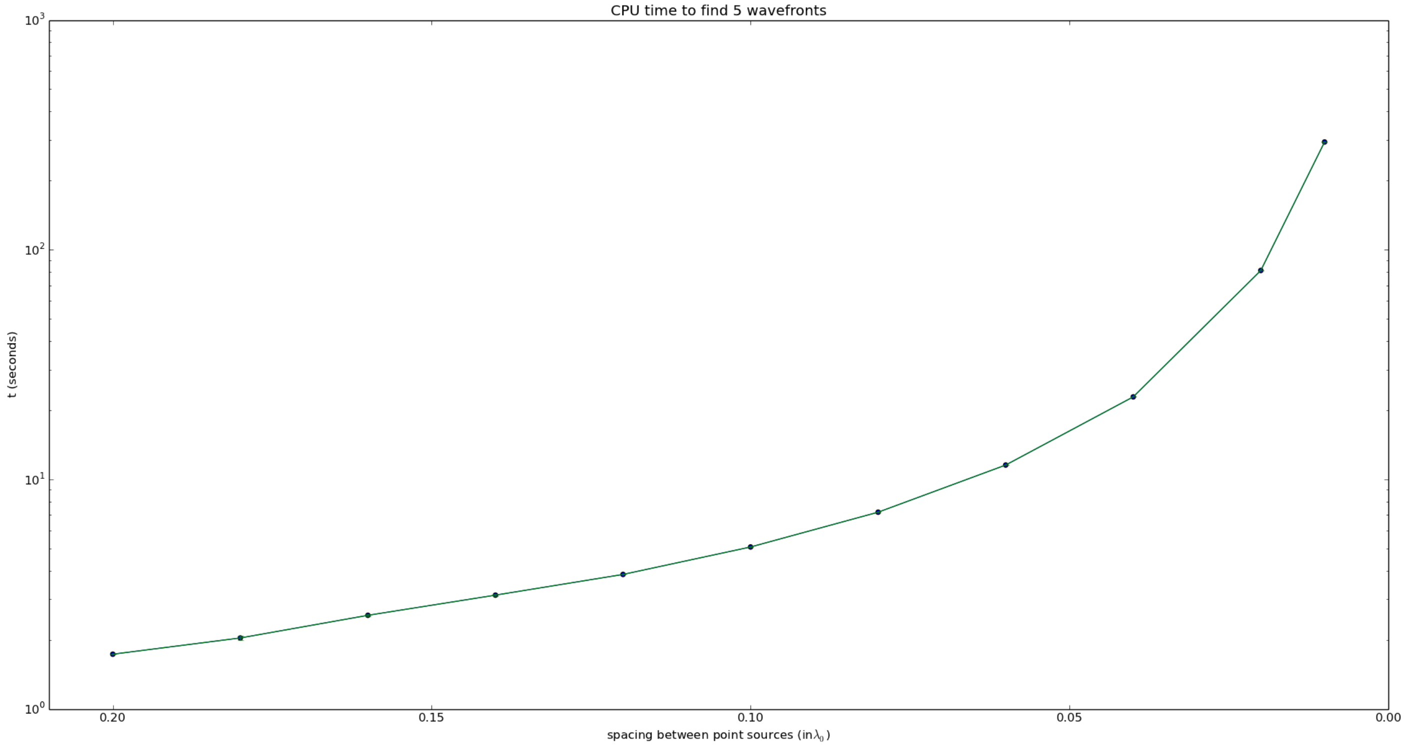


Strongly focused Gaussian beam, intensity



Phase-jump issues expected to be solved by improved (adaptive?) definition of interval for zero search by bisection method.

CPU time



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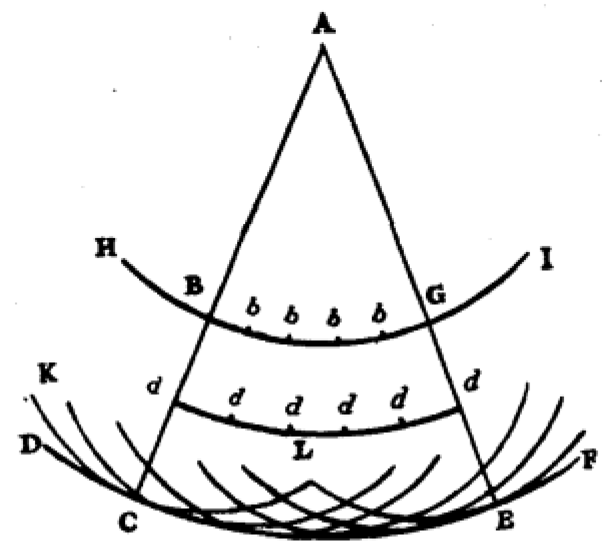
Summary and future work

- Numerical deployment of Huygens' principle [*Traitė de la Lumiere*, 1690] is surprisingly rare.
- Some exceptions in seismic wave research, and e.m. wave outreach/teaching.
- Here we started exploring its applicability as a potential new method for wave propagation in plasmas, including diffraction.
- Very simple cases successfully modeled in 2D.
  - Diffraction around obstacle.
  - Refraction in slowly varying medium.
  - Gaussian beams.



- Minor issues of phase-jumps and corrugations currently being fixed.
- Formulated extension to 3D anisotropic media such as magnetized plasmas.
- Future work: (1) detailed study of cpu-time scaling, (2) anisotropy, (3) comparison with ray tracing, full-wave and PIC, (4) extension to 3D.