Modeling Of Tokamak Plasmas

J.D. Callen, University of Wisconsin, Madison, WI 53706-1609 USA

Poster P1.040, 2015 Sherwood Fusion Theory Conference, New York University, March 16-18, 2015

Questions To Be Addressed:[1]

1) What effects need to be modeled for ITER plasmas?

2) What equations need to be solved for integrated simulations?

3) How can MHD, kinetic and transport models be self-consistent?

Outline:

- **Modeling ITER plasmas involves disparate $\vec{x}, t$ scales and issues.**
- **Comprehensive modeling of plasmas involves some key elements:**
  plasma kinetic equation with Fokker-Planck collision operator and sources, fluid equations when $t > 1/\nu$, Chapman-Enskog (C-E) kinetic equation, extended MHD (ideal MHD for $t < 1/\nu$, neoclassical MHD for $t > 1/\nu$), collisional-, fluctuation- and 3-D- induced fluxes via C-E kinetic equation, and comprehensive plasma transport equations for n_e, $\Omega_{\rm tor}$, p_e, p_i and $\psi_{\rm p}$.
- **Need modular approach for integrated modeling of ITER plasmas.**

[1] J.D. Callen, CEMRACS 2014 "Fluid and transport modeling of plasmas" lectures available via `http://homepages.cae.wisc.edu/~callen/plasmas`.

This Evolving Study Has A Long-Term Objective

- **Develop a strategic vision for how the major plasma models (MHD, kinetics, transport) that operate on a hierarchy of sequentially longer time scales can and should be combined into self-consistent, comprehensive, integrated simulations of plasmas in present tokamaks to develop a "predictive capability" for ITER:**

  what physical effects should be included?,

  what equations need to be solved?,

  how can they be coupled and made self-consistent?

- **This presentation will:**

  discuss logic of combining MHD, kinetics, transport approaches,

  highlight effects of diamagnetic-level flows and small 3-D fields,

  focus on descriptions inside separatrix, but can be generalized.

Characteristic Length And Time Scales In Plasmas Span Many Orders of Magnitude (ITER and ICF)

- **Projected parameters for ITER where $B_t = 5.6$ T, $T_e \sim T_i \sim 10$ keV, $n_e \sim 10^{20}$ m^{-3}, and major/mid-plane minor radius $\simeq$ 6 m/2 m are:**

Length Scales

minimum impact distance	$b_{\min}^{\rm qm}$	10^{-12} m
mean particle spacing	$n_e^{-1/3}$	2×10^{-7} m
Debye shielding length	λ_{De}	7×10^{-5} m
deuteron gyroradius	ϱ_D	3×10^{-3} m
average minor radius	$\bar{a}$	3 m
collision length	λ_e	1.2×10^4 m

Time Scales

plasma period	$1/\omega_p$	2×10^{-12} s
deuteron gyroperiod	$1/\omega_{cD}$	3×10^{-9} s
Alfvén period	$\bar{a}/c_{\rm A}$	5×10^{-7} s
sound wave period	$\bar{a}/c_S$	4×10^{-6} s
electron collision time	$1/\nu_e$	2×10^{-4} s
deuteron collision time	$1/\nu_D$	3×10^{-2} s
energy confinement time	τ_E	6 s
fusion collision time	$1/\nu_{\rm fus}$	200 s
magnetic field diffusion	τ_R	1000 s

Dimensionless Parameters	formula	ICF	ITER
number of electrons in a Debye cube	$n_e\lambda_{De}^3$	3.5×10^3	4×10^7
electron collision to plasma frequency	ν_e/ω_p	4×10^{-5}	10^{-8}
ϱ_*: deuteron gyroradius to average minor radius	$\varrho_D/\bar{a}$		10^{-3}
collision length to relevant length	λ_e/L	$\lambda_e/\bar{a} \sim 0.4$	$\lambda_e/2\pi R = 320$

Integrated Simulations Should Include Many Effects

- **Current ramp-up from breakdown to current flattop, smooth turnoff:**
  tearing-limited resistive evolution of $\vec{J}(\rho, t)$; NBI torque to avoid locked modes.

- **MHD stability boundaries and tearing-induced topology changes:**
  low q_{95} and high β_N stability limits; slowly growing tearing-induced islands.

- **Use of 3-D fields to control plasma rotation and H-mode pedestals:**
  NTV control of toroidal plasma rotation; RMPs to mitigate/suppress ELMs.

- **Heating, momentum, particle and poloidal flux sources and sinks:**
  core NBI, ICRF, EC; edge neutral recycling, charge-exchange, radiation etc.

- **Transport evolution and control of n_e, $\Omega_{\rm tor}(E_\rho)$, T_e, T_i, $\psi_{\rm p}$ profiles:**
  equil. & stiff ones, internal & H-mode transport barriers, hybrid flux pumping.

All these effects need to be validated and modeled self-consistently to develop a predictive capability for ITER plasmas.

MHD, Kinetics, Transport Are Usually Treated Separately

- **MHD models** can provide
  axisymmetric equilibrium — force balances, $P = P(\psi_{\rm p})$, $\vec{B}_0 = I\vec{\nabla}\zeta + \vec{\nabla}\zeta\times\vec{\nabla}\psi_{\rm p}$;
  ideal MHD stability constraints — minimum $q(\psi_{\rm p})$, maximum β;
  neoclassical equilibrium flows — $\|$ Ohm's law with bootstrap current, $V_{{\rm pol}\,i}$;
  magnetic reconnection of field lines at rational surfaces in dissipative MHD — tearing instabilities, penetration of external $\delta\vec{B}$s, locked modes; and
  plasma amplification of external 3-D $\delta\vec{B}$'s that couple to least stable modes.

- **Kinetic models** can provide transport fluxes in $\vec{B}_0 + \delta\vec{B}$ due to
  Coulomb collisional effects — classical, neoclassical, paleoclassical;
  fluctuation-induced microturbulence — ITG, TEM, KBM etc.; and
  small 3-D magnetic fields — error fields, ripple, NTV, RMPs, flutter, islands.

- Plasma **transport models** are based on axisym. $\vec{B}_0$ surfaces and use
  fluxes from kinetics plus sources and sinks in transport evolution equations
  to determine the ρ, t behavior of plasma parameters on transport time scale.

Comprehensive integrated modeling of tokamak plasmas needs a self-consistent theoretical framework for combining these models.

Fundamental Equation Is Plasma Kinetic Equation (PKE)

- **PKE is** $\boxed{\dfrac{df_s(\vec{x},\vec{v},t)}{dt} \equiv \dfrac{\partial f_s}{\partial t} + \vec{v}\cdot\dfrac{\partial f_s}{\partial \vec{x}} + \dfrac{\vec{F}_s}{m_s}\cdot\dfrac{\partial f_s}{\partial \vec{v}} \;=\; \mathcal{C}\{f_s\} + \mathcal{S}\{f_s\},}$ in which

  $f_s \equiv f_s(\vec{x},\vec{v},t)$ is the distribution function in 6-D $(\vec{x},\vec{v})$ phase space,

  $\vec{F}_s \equiv q_s(\vec{E} + \vec{v}\times\vec{B})$ is the Lorentz force on a charged particle,

  $\mathcal{C}\{f_s\}$ is the Fokker-Planck Coulomb collision operator which is local in $\vec{x}$, and

  $\mathcal{S}\{f_s\}$ is source operator that represents heating, current-drive, c-x etc. effects.

- While the PKE is fundamental, the 3-D $(\vec{x})$ fluid moment equations for n_s, $\vec{V}_s$, p_s are

  feasible, appropriate, useful and needed,

  exact to extent that relevant Chapman-Enskog kinetic equation (CEKE, p 12,13) can be solved for kinetic distortion $F_s \equiv f_s - f_{\mathrm{Max}\,s}(\vec{x},\vec{v},t)$,

  which then yields the needed closure and collisional moments, and

  provide the basis for both extended MHD and plasma transport equations.

Species s Fluid Moment Equations Are Useful, Needed

- The $\int d^3v\,(1, m_s\vec{v}, m_s v^2/2)$ moments of the plasma kinetic equation (PKE) yield the species s *complete fluid moment equations*:

$$\text{density} \qquad (\partial/\partial t + \vec{V}_s\cdot\vec{\nabla})\, n_s \;=\; -\,n_s\vec{\nabla}\cdot\vec{V}_s + S_{ns},$$

$$\text{momentum} \quad m_s n_s(\partial/\partial t + \vec{V}_s\cdot\vec{\nabla})\,\vec{V}_s \;=\; n_s q_s(\vec{E} + \vec{V}_s\times\vec{B}) - \vec{\nabla}p_s - \vec{\nabla}\cdot\overleftrightarrow{\pi}_s + \vec{R}_s + \vec{S}_{\vec{p}s},$$

$$\text{energy} \qquad \frac{3}{2}(\partial/\partial t + \vec{V}_s\cdot\vec{\nabla})\,p_s \;=\; -\,\frac{5}{2}p_s\vec{\nabla}\cdot\vec{V}_s + p_s\,\dot{s}_{\mathrm{M}s}, \quad \text{or},$$

$$\text{entropy} \qquad (\partial/\partial t + \vec{V}_s\cdot\vec{\nabla})\,s_{\mathrm{M}s} \;=\; \dot{s}_{\mathrm{M}s} \equiv (-\vec{\nabla}\cdot\vec{q}_s - \overleftrightarrow{\pi}_s : \vec{\nabla}\vec{V}_s + Q_s + S_{\mathcal{E}s})/p_s,$$

in which the species s isotropic-Maxwellian-based collisional entropy is

$$s_{\mathrm{M}s}(\vec{x},t) \equiv -\frac{1}{n}\int d^3v\, f_{\mathrm{iM}s}\ln f_{\mathrm{iM}s} = \frac{3}{2}\ln\left(\frac{p_s}{n^{5/3}}\right) + C, \quad \textit{collisional entropy}.$$

- But equations are incomplete until these "closures" are specified:

stress $\overleftrightarrow{\pi}_s$, heat flux $\vec{q}_s$; collisional friction force $\vec{R}_s$, energy exchange Q_s.

Modeling Tokamaks Employs Some Key Approximations

- **The fundamental approximation used in modeling tokamak plasmas is that the gyroradii ($\varrho_i \equiv v_{Ti}/\omega_{ci}$) of the charged ions in the magnetic field $\vec{B}$ are small compared to macroscopic gradient scale lengths $L \sim 1/|\vec{\nabla} \ln(B, n, T)|$, i.e., $\varrho_* \equiv \varrho_i/L \ll 1$. This facilitates analyses on a hierarchy of ever longer time scales:**

  ϱ_*^0: **parallel (to $\vec{B}$) guiding center motion, Alfvén and sound waves ($\sim \mu$s),**

  ϱ_*^1: **particle drifts across the magnetic field, fluid descriptions, collisional effects on species flows in flux surfaces, magnetic reconnection ($\gtrsim$ ms), and finally**

  ϱ_*^2: **transport of plasma across magnetic field lines and flux surfaces due to collisions, radially localized microturbulence and 3-D field effects ($\gtrsim$ s).**

- **Sequential ϱ_* orders cause momentum balance component effects:**

  ϱ_*^0, $\vec{\nabla}\psi_{\rm p}\,\cdot \Rightarrow$ **MHD comp. Alfvén waves enforce ($\sim 0.4\,\mu$s) <u>radial force balances</u>,**

  ϱ_*^1, $\vec{B}_0\,\cdot \Rightarrow$ **drifts, neoclassical MHD <u>flows within $\psi_{\rm p}$ surfaces</u> (e 0.2 ms, i 30 ms),**

  ϱ_*^2, $\vec{e}_\zeta\,\cdot \Rightarrow$ **toroidal torques on plasma cause <u>radial transport fluxes</u> ($\tau_E \sim 6$ s).**

Extended MHD Model Includes Ideal MHD And The Dissipative Effects Of Closure And Collisional Moments

- **The extended MHD equations for a magnetized plasma are obtained by summing the fluid moment equations over species. Together with equations for the magnetic field they are**

Extended MHD plasma description (for ideal MHD $\vec{R}_e$, $\overleftrightarrow{\Pi}$, $\overleftrightarrow{\pi}_e$, $\sum_s \dot{s}_{\mathrm{M}s} \to 0$):

mass density $\quad (\partial/\partial t + \vec{V}\cdot\vec{\nabla})\,\rho_m = -\,\rho_m \vec{\nabla}\cdot\vec{V},$

charge continuity $\quad \vec{\nabla}\cdot\vec{J} = 0,$

momentum $\quad \rho_m(\partial/\partial t + \vec{V}\cdot\vec{\nabla})\vec{V} = \vec{J}\times\vec{B} - \vec{\nabla}P - \vec{\nabla}\cdot\overleftrightarrow{\Pi},$

Ohm's law $\quad \vec{E} = -\,\vec{V}\times\vec{B} + \vec{R}_e/n_e e + (\vec{J}\times\vec{B} - \vec{\nabla}p_e - \vec{\nabla}\cdot\overleftrightarrow{\pi}_e)/n_e e,$

equation of state $\quad (\partial/\partial t + \vec{V}\cdot\vec{\nabla})\ln(P/\rho_m^{5/3}) = \sum_s \dot{s}_{\mathrm{M}s}.$

Maxwell equations for extended MHD (no Gauss' law, $\vec{E}$ from Ohm's law):

Faraday's law $\quad \partial\vec{B}/\partial t = -\,\vec{\nabla}\times\vec{E},$

no magnetic monopoles $\quad \vec{\nabla}\cdot\vec{B} = 0,$

nonrelativistic Ampere's law $\quad \vec{J} = \vec{\nabla}\times\vec{B}/\mu_0.$

Extended MHD Equations Produce Many Effects

- **Ideal MHD in a lowest order axisymmetric tokamak provides**

  equilibrium radial force balance that is enforced by compressional Alfvén waves on $\bar{a}/c_{\mathrm{A}} \sim 0.5\,\mu$s time scale and yields Grad-Shafranov equation for ψ_{p},
  frozen flux theorem ($\vec{B}$ advects with $\vec{V}_\perp$) due to $\vec{E} + \vec{V}\times\vec{B} = \vec{0}$ Ohm's law,
  ideal MHD constraints for kink (min q), $\vec{\nabla}P$-driven (max β) stability, and
  amplification of driven 3-D perturbations that couple to least stable modes.

- **Two-fluid MHD causes a diamagnetic-level flow constraint because**

  $\vec{\nabla}\psi_{\mathrm{p}}\,\cdot$ its Ohm's law $\vec{E}+\vec{V}\times\vec{B} = \vec{\nabla}p_i/n_iq_i$ yields relation between flows in surface

$$\Omega_{\mathrm{tor}} \equiv \vec{V}_i\cdot\vec{\nabla}\zeta = -\left(\frac{\partial\Phi_0}{\partial\psi_{\mathrm{p}}} + \frac{1}{n_iq_i}\frac{\partial p_i}{\partial\psi_{\mathrm{p}}}\right) + q\,\vec{V}_i\cdot\vec{\nabla}\theta \implies V_{\mathrm{tor}} \simeq \frac{E_\rho}{B_{\mathrm{p}}} - \frac{1}{n_iq_iB_{\mathrm{p}}}\frac{dp_i}{d\rho} + \frac{B_{\mathrm{t}}}{B_{\mathrm{p}}}V_{\mathrm{pol}\,i}\,.$$

- **Extended MHD with collision-based closures for $t > 1/\nu$ leads to**

  reconnection of field lines in dissipative singular layers at rational surfaces,
  classical and neoclassical tearing-type instabilities $\implies$ magnetic islands,
  FSA $\parallel$ neo Ohm's law $\langle\vec{B}_0\cdot\vec{E}^{A}\rangle = \eta_\parallel^{\mathrm{nc}}\Big[\langle\vec{B}_0\cdot\vec{J}\rangle - \langle\vec{B}_0\cdot\vec{J}_{\mathrm{drives}}\rangle\Big]$, for $t > 0.2$ ms,
  poloidal ion flow damped to $V_{\mathrm{pol}\,i}^{\mathrm{nc}} \simeq (c_{\mathrm{p}}/q_iB)\,(dT_i/d\rho)$, $c_{\mathrm{p}} \simeq 1.17$, for $t > 30$ ms,
  3-D $\vec{B}$ field modifications and their effects — error field locking, NTV, RMPs.

Extended MHD Constraints Should Be Used In Kinetics

- **Lowest order constraints for MHD stable plasmas:**

  ϱ_*^0: axisymmetric magnetic field $\vec{B}_0 \equiv I\vec{\nabla}\zeta + \vec{\nabla}\zeta\times\vec{\nabla}\psi_{\rm p} = \vec{\nabla}\psi_{\rm p}\times\vec{\nabla}(q\,\theta - \zeta)$, with poloidal magnetic flux surfaces $\psi_{\rm p}$ determined from Grad-Shafranov equation,

  ϱ_*^1: a consistency relation between the flows of $V_{\rm tor} \simeq \frac{E_\rho}{B_{\rm p}} - \frac{1}{n_i q_i B_{\rm p}}\frac{dp_i}{d\rho} + \frac{B_{\rm t}}{B_{\rm p}} V_{{\rm pol}\,i}$ — which does not determine the electric field E_ρ or plasma toroidal rotation frequency $\Omega_{\rm tor} \simeq V_{\rm tor}/R$, but provides a relation between them.

- **First order (ϱ_*^1) extended MHD "equilibrium effects" for tearing-mode stable plasmas that could be used to speed convergence:**

  Equilibrium Maxwellian distribution could include the following ϱ_* flows:

  parallel electron flow that yields the FSA $\|$ neoclassical Ohm's law, and

  the neoclassical poloidal ion flow $V_{{\rm pol}\,i}^{\rm nc}$.

  A magnetic field structure $\vec{B} = \vec{B}_0 + \delta\vec{B}$ in which the ϱ_* perturbations $\delta\vec{B}$

  are nonzero in dissipative singular layers near rational surfaces, and

  are driven in plasma outside them by small externally-imposed 3-D fields.

Use Chapman-Enskog Approach For Kinetic Equation

- **Chapman-Enskog Ansatz posits species distribution function has two parts — "dynamic" space- and time-dependent Maxwellian $f_{\rm M}$ with parameters $n(\vec{x},t), \vec{V}(\vec{x},t), T(\vec{x},t)$ plus kinetic distortion F:**

$$f(\vec{x},\vec{v},t) = f_{\rm M}(\vec{x},\vec{v},t) + F(\vec{x},\vec{v},t), \quad \text{in which dynamic Maxwellian is}$$

$$f_{\rm M}(\vec{x},\vec{v},t) = f_{\rm M}[n(\vec{x},t), \vec{V}(\vec{x},t), T(\vec{x},t), \vec{v}] = \frac{n(\vec{x},t)\, e^{-m\,[\vec{v}-\vec{V}(\vec{x},t)]^2/2T(\vec{x},t)}}{[2\pi\, T(\vec{x},t)/m]^{3/2}}.$$

- **Substituting this Ansatz into the plasma kinetic equation yields**

$$\boxed{\frac{dF}{dt} - \mathcal{C}\{f\} - \mathcal{S}\{f\} = -\frac{df_{\rm M}}{dt},}$$ ***general Chapman-Enskog kinetic equation***,

$$\begin{aligned}
\frac{df_{\rm M}}{dt} = \quad f_{\rm M}\Bigg[& -\frac{1}{p}\,\vec{v}_r\cdot\left(nq\,[\vec{E}+\vec{V}\times\vec{B}] - \vec{\nabla}p - mn\,(\frac{\partial}{\partial t}+\vec{V}\cdot\vec{\nabla})\vec{V}\right) && \text{forces}\\
& + \left(\frac{mv_r^2}{2T}-\frac{5}{2}\right)\frac{1}{T}\,\vec{v}_r\cdot\vec{\nabla}T && T\ \text{gradient}\\
& + \frac{m}{T}\left(\vec{v}_r\vec{v}_r - \frac{v_r^2}{3}\mathsf{I}\right):\mathsf{W},\ \ \mathsf{W}\equiv\frac{1}{2}[\vec{\nabla}\vec{V}+(\vec{\nabla}\vec{V})^{\mathsf{T}}-\frac{2}{3}\mathsf{I}\,\vec{\nabla}\cdot\vec{V}] && \text{rate of strain}\\
& + \frac{S_n}{n} + \left(\frac{mv_r^2}{2T}-\frac{3}{2}\right)\left(\frac{2\,\dot{s}_{\rm M}}{3\,p}-\frac{S_n}{n}\right)\Bigg]. && \text{xport sources}
\end{aligned}$$

Chapman-Enskog Approach Is Useful And Important

- **Chapman-Enskog kinetic equation (CEKE)** on preceding page is still exact since no approximations or truncations have been made.

- Lowest order drift-kinetic CEKE obtained by using momentum equation from p 7 and gyro-averaging yields ($\hat{\mathrm{b}} \equiv \vec{B}/B_0$)[2,3,4]

$$\frac{\partial \overline{F}}{\partial t} + (v_\parallel \hat{\mathrm{b}} + \vec{v}_d) \cdot \frac{\partial \overline{F}}{\partial \vec{x}} - \overline{\mathcal{C}\{f_{\mathrm{M}} + F\}} - \overline{\mathcal{S}\{f_{\mathrm{M}} + F\}}$$
$$= \overline{f_{\mathrm{M}}} \left[\left(\frac{mv^2}{2T} - \frac{5}{2} \right) v_\parallel \hat{\mathrm{b}} \cdot \vec{\nabla} \ln T + \frac{m}{T} \left(v_\parallel^2 - \frac{v_\perp^2}{2} \right) [\, \hat{\mathrm{b}} \cdot (\hat{\mathrm{b}} \cdot \vec{\nabla}) \overline{\vec{V}} - \vec{\nabla} \cdot \overline{\vec{V}}/3] - \frac{v_\parallel}{p} \hat{\mathrm{b}} \cdot [\vec{\nabla} \cdot \overline{\overleftrightarrow{\pi}_\parallel} - \vec{R}] \right].$$

  A more general gyrokinetic version is needed to include $k_\perp \varrho_i \sim 1$ physics.

- Since by construction $\int d^3v \, (1, \vec{v}, v_r^2) \, F = 0$,

  the kinetic distortion F does not produce any extraneous δn, $\delta \vec{V}$ or δp.

  Hence it produces no $\delta \vec{J}$, which is consistent with extended MHD since $\vec{J} = \vec{\nabla} \times \vec{B}$ is "owned" by the Maxwell and MHD equations.

- The $\vec{q}_s$, $\overleftrightarrow{\pi}_s$, $\vec{R}_s$, Q_s velocity-space closure and collisional moments

$$\vec{q} \equiv \int d^3v \, \vec{v} \left(\frac{mv^2}{2T} - \frac{5}{2} \right) \overline{F}, \quad \overleftrightarrow{\pi} \equiv \int d^3v \, m(\vec{v}\vec{v} - v^2 \mathsf{I}/3) \overline{F}, \quad \vec{R} \equiv \int d^3v \, m \vec{v} \, \mathcal{C}\{\overline{F}\}, \quad Q \equiv \int d^3v \frac{mv^2}{2} \mathcal{C}\{\overline{F}\}$$

  will be consistent with extended MHD and transport equations.

[2] Z. Chang and J.D. Callen, "Unified fluid/kinetic description of plasma micronstabilities. Part I," Phys. Fluids B **4**, 1167 (1992).
[3] S. Bruner, E. Valeo, J.A. Krommes, "Collisional delta-f scheme with evolving background for transport time scale simuls," Phys. Pl. **6**, 4504 (1999).
[4] J.J. Ramos, Phys. Pl. **17**, 082502 (2010); J.J. Ramos, Phys. Pl. **18**, 102506 (2011); B.C. Lyons, S.C. Jardin, J.J. Ramos, Phys. Pl. **19**, 082515 (2012).

There Can Be Inconsistency Issues With Gyrokinetics

- **The gyrokinetic formalism, in its usual implementations, is inconsistent with extended MHD and transport equations because**

  it is based on the reduced MHD axisymmetric magnetic field representation which neglects compressional Alfvén wave constraints on $V_{\rm tor}$ and evolving $\vec{B}$,

  its "drive" terms are of the form $\vec{v}_{\rm d} \cdot \vec{\nabla} f_{\rm Max}$ which is not consistent with Chapman-Enskog-type drives [$\sim \vec{v} \cdot (nq\vec{E} - \vec{\nabla} p)$, $\vec{v} \cdot \vec{\nabla} T$, $(\vec{v}\vec{v} - (v^2/3)\mathsf{I}) : \mathsf{W}$];

  thus, its "input" does not include many $V_{\rm tor}$, 3-D $\delta\vec{B}$ effects and its "output fluxes" will be inconsistent with the tokamak plasma transport equations.

- **Most gyrokinetic studies concentrate on microturbulence & fluxes caused by particle drifts across axisymmetric surfaces and they**

  employ a large $V_{\rm tor} \simeq E_\rho / B_{\rm p}$ approximation plus diamagnetic flow but neglect the ion poloidal flow in the radial force balance ($V_{\rm tor}$) equation,

  concentrate on high n modes and do not treat low n global modes well because such modes require $\vec{J} = \vec{\nabla} \times \vec{B}/\mu_0$ with $\partial \vec{B}/\partial t = \vec{\nabla} \times \vec{E}$ in which $\vec{E}$ is from Ohm's law plus there are dissipative layer effects at rational surfaces,

  treat non-ambipolar transport fluxes inconsistently — magnetic flutter determines local E_ρ but many other non-ambipolar transport effects (e.g., $\perp$ viscous, 3-D and NBI torques) are neglected in E_ρ determination.

Tokamak Plasma Transport Equations Include Many Effects

- **Transport equations for n_e, $L_{\rm tor} \equiv \rho_m \langle R^2 \rangle \Omega_{\rm tor}$, p_s, $\psi_{\rm p}$ with sources:**[5]

density
$$\frac{1}{V'}\frac{\partial}{\partial t}\Big|_{\psi_{\rm p}} n_e V' + \dot{\rho}_{\psi_{\rm p}} \frac{\partial n_e}{\partial \rho} + \frac{1}{V'}\frac{\partial}{\partial \rho}(V' \Gamma) = \langle \overline{S}_n \rangle,$$

toroidal momentum
$$\frac{1}{V'}\frac{\partial}{\partial t}\Big|_{\psi_{\rm p}} L_{\rm tor} V' + \dot{\rho}_{\psi_{\rm p}} \frac{\partial L_{\rm tor}}{\partial \rho} + \frac{1}{V'}\frac{\partial}{\partial \rho}(V' \overline{\Pi}_{\rho\zeta}) = \langle \vec{e}_\zeta \cdot (\overline{\vec{J}\times\vec{B}} - \vec{\nabla}\cdot \overline{\overleftrightarrow{\Pi}} + \overline{\vec{S}}_{\vec{p}}) \rangle,$$

energy
$$\frac{3}{2} p_s \frac{\partial}{\partial t}\Big|_{\psi_{\rm p}} \ln p_s V'^{5/3} + \frac{3}{2} \dot{\rho}_{\psi_{\rm p}} \frac{\partial p_s}{\partial \rho} + \frac{1}{V'}\frac{\partial}{\partial \rho}(V' \Upsilon_s) + \langle \vec{\nabla}\cdot \vec{q}^{\,\rm pc}_{*s} \rangle = \overline{Q}_{{\rm net}\, s},$$

poloidal flux
$$\frac{\partial \psi_{\rm p}}{\partial t}\Big|_{\psi_{\rm t}} = D_\eta\, \Delta^+ \psi_{\rm p} - S_{\psi_{\rm p}}, \quad D_\eta \equiv \frac{\eta_\parallel^{\rm nc}}{\mu_0}, \quad S_{\psi_{\rm p}} = \frac{\partial \Psi_{\rm p}}{\partial t} + \frac{\eta_\parallel^{\rm nc}}{I \langle R^{-2} \rangle} \langle \vec{B}_0 \cdot \vec{J}_{\rm drives} \rangle.$$

- **There are many classes of effects in plasma transport equations:**

  transients in the poloidal flux $\psi_{\rm p}$ via $\partial/\partial t|_{\psi_{\rm t}}$ and advection of $\psi_{\rm p}$ surfaces relative to the toroidal-flux-based radial coordinate ρ via $\dot{\rho}_{\psi_{\rm p}} \equiv \dot{\psi}_{\rm p}/\psi'_{\rm p}$,

  transport fluxes of ambipolar density Γ, total momentum $\overline{\Pi}_{\rho\zeta}$ and heat Υ_s, $\vec{q}^{\,\rm pc}_{*s}$ "radially" across $\psi_{\rm p}$ poloidal flux surfaces that have many contributions induced by collisions, microturbulence and 3-D effects on each species s,

  toroidal ($\vec{e}_\zeta \equiv R^2 \vec{\nabla}\zeta$) plasma torques caused by $\overline{\vec{J}\times\vec{B}}$ and viscous stresses $\vec{\nabla}\cdot\overline{\overleftrightarrow{\Pi}}$,

  ambipolar density $\langle \overline{S}_n \rangle$, toroidal momentum $\langle \vec{e}_\zeta \cdot \overline{\vec{S}}_{\vec{p}} \rangle$ and energy $\langle \overline{S}_{\mathcal{E}} \rangle$ sources that contribute to the net energy heating rate $\overline{Q}_{{\rm net}\,s}$, which includes Joule and external heating sources (NBI, ICRH, ECH etc.) plus radiation losses.

[5] J.D. Callen, C.C. Hegna, and A.J. Cole, "Transport equations in tokamak plasmas," Phys. Plasmas **17**, 056113 (2010).

Plasma Toroidal Rotation Equation Includes 3-D Effects

- The magnetic field will be represented in $\psi_\mathrm{p}, \theta, \zeta$ coordinates by

$$\vec{B} = \underbrace{\vec{B}_0(\psi_\mathrm{p},\theta)}_{\text{2D, axisymm.}} + \sum_{n,m\neq 0} \underbrace{\delta\vec{B}_n(\psi_\mathrm{p},m)\cos(m\theta - n\zeta - \varphi_{m,n})}_{\text{low } m,n \text{ resonant, non-resonant}} + \underbrace{\delta\vec{B}_N(\psi_\mathrm{p},\theta)\cos(N\zeta)}_{\text{medium } n\text{, ripple}} + \underbrace{\cdots}_{\mu\text{turb.}} .$$

- On μs time scale compressional Alfvén waves enforce radial force balance:

$$\Omega_\mathrm{tor} \equiv \vec{V}_i \cdot \vec{\nabla}\zeta = -\left(\frac{\partial \Phi_0}{\partial \psi_\mathrm{p}} + \frac{1}{n_i q_i}\frac{\partial p_i}{\partial \psi_\mathrm{p}}\right) + q\,\vec{V}_i \cdot \vec{\nabla}\theta \quad \Longrightarrow \quad V_\mathrm{tor} \simeq \frac{E_\rho}{B_\mathrm{p}} - \frac{1}{n_i q_i B_\mathrm{p}}\frac{dp_i}{d\rho} + \frac{B_\mathrm{t}}{B_\mathrm{p}} V_{\mathrm{pol}\,i}.$$

- On the ms time scale poloidal flow is damped to $V_{\mathrm{pol}\,i}^{\mathrm{nc}} \simeq (c_\mathrm{p}/q_i)(dT_i/d\psi_\mathrm{p})$.

- Toroidal plasma torques cause radial particle fluxes: $\vec{e}_\zeta \cdot \vec{F}_\mathrm{orce} = -\,q_s \vec{\Gamma}_s \cdot \vec{\nabla}\psi_\mathrm{p}$.

- Setting the total radial plasma current induced by sum of the non-ambipolar particle fluxes to zero yields transport equation[5,6] for plasma toroidal angular momentum density $L_\mathrm{tor} \equiv \sum_\mathrm{ions} m_i n_i \langle R^2 \vec{V}_i \cdot \vec{\nabla}\zeta\rangle$, $\boxed{\Omega_\mathrm{tor}(\rho,t) \equiv L_\mathrm{tor}/m_i n_i \langle R^2\rangle :}$

$$\boxed{\underbrace{\frac{\partial L_\mathrm{tor}}{\partial t}}_{\text{inertia}} \simeq -\underbrace{\langle \vec{e}_\zeta \cdot \vec{\nabla} \cdot \overleftrightarrow{\pi}_{i\parallel}^{\,3\mathrm{D}} \rangle}_{\text{NTV from } \delta B} + \underbrace{\langle \vec{e}_\zeta \cdot \overline{\delta\vec{J} \times \delta\vec{B}} \rangle}_{\text{resonant FEs}} - \underbrace{\langle \vec{e}_\zeta \cdot \vec{\nabla} \cdot \overleftrightarrow{\pi}_{i\perp} \rangle}_{\text{cl, neo, paleo}} - \underbrace{\frac{1}{V'}\frac{\partial}{\partial \rho}(V' \Pi_{i\rho\zeta})}_{\text{Reynolds stress}^6} + \underbrace{\langle \vec{e}_\zeta \cdot \textstyle\sum_s \vec{S}_{\vec{p}s} \rangle}_{\text{mom. sources}} .}$$

- Radial electric field for net ambipolar transport is determined by Ω_tor (from L_tor):

$$\boxed{E_\rho \equiv -\,|\vec{\nabla}\rho|\,\partial\Phi_0/\partial\rho \simeq |\vec{\nabla}\rho|\,[\,\Omega_\mathrm{tor}\,\psi_\mathrm{p}' + (1/n_{i0}q_i)\,dp_i/d\rho - (c_\mathrm{p}/q_i)\,dT_i/d\rho],} \qquad \psi_\mathrm{p}' \simeq RB_\mathrm{p}.$$

[6] J.D. Callen, A.J. Cole, C.C. Hegna, "Toroidal flow and radial paricle flux in tokamak plasmas," Phys. Pl. **16**, 082504 (2009); Err. **20**, 069901 (2013) .

3-D Field Effects In Toroidal Momentum Equation

- General transport equation for toroidal angular momentum density $L_{\rm tor} \equiv \rho_m \langle R^2 \rangle \Omega_{\rm tor}$ is[6]

toroidal momentum $$\frac{1}{V'}\frac{\partial}{\partial t}\Big|_{\psi_{\rm p}} L_{\rm tor} V' + \dot{\rho}_{\psi_{\rm p}} \frac{\partial L_{\rm tor}}{\partial \rho} + \frac{1}{V'}\frac{\partial}{\partial \rho}(V' \overline{\Pi}_{\rho\zeta}) = \langle \vec{e}_\zeta \cdot \Big(\overline{\vec{J}\times\vec{B}} - \vec{\nabla}\cdot \overline{\overleftrightarrow{\Pi}} + \overline{\vec{S}}_{\vec{p}} \Big) \rangle,$$

- Small 3-D field ($|\delta\vec{B}|/B_0 \sim \varrho_*$) effects come about in many ways:[7]

externally applied resonant $m/n \simeq q$ and non-resonant fields cause field error (FE, $\langle \vec{e}_\zeta \cdot \overline{\vec{J}\times\vec{B}} \rangle$) and neo. toroidal viscous (NTV, $\langle \vec{e}_\zeta \cdot \vec{\nabla}\cdot \overline{\overleftrightarrow{\Pi}_\parallel} \rangle$) damping of $\Omega_{\rm tor}$,

toroidal magnetic field "ripple" $\hat{\rm b}_0 \cdot \delta\vec{B}_N$ caused by the finite number of coils that produce the toroidal magnetic field, which damps $\Omega_{\rm tor}$ via NTV,

externally applied edge resonant magnetic perturbations (RMPs) used to modify the pressure profile there and stabilize edge MHD instabilities, and

spontaneous magnetic perturbations in the plasma which are caused by extended MHD macroscopic plasma instabilities that are controlled, e.g., neoclassical tearing modes (NTMs) or resistive wall modes (RWMs).

[7] J.D. Callen, topical review on "Effects of 3D magnetic perturbations on torodal plasmas," Nucl. Fusion **51**, 094026 (2013).

Differences From Present Approaches Are Suggested

- **The fluid moment equations should be accepted as the basis for self-consistent and comprehensive modeling of tokamak plasmas.**

- **Extended MHD should be used to ensure macroscopic stability, and provide ϱ_* order flows in flow-shifted Maxwellian equilibrium plus evolving $\vec{B}$ field for Chapman-Enskog-based kinetic analysis.**

- **Gyrokinetic community should accept that its role is not to "do everything" but use flows and $\vec{B}$ from extended MHD as "input" and produce closure moments for transport equations as "output."**

- **The tokamak plasma transport equations for electron density n_e, plasma toroidal rotation frequency $\Omega_{\rm tor}(E_\rho)$, species pressures p_s and poloidal flux $\psi_{\rm p}$ should all be solved for simultaneously, i.e., $\Omega_{\rm tor}(E_\rho)$ should not just be obtained from experimental data.**

- **This modular approach needs to be iterated in order to obtain self-consistency between the MHD, kinetic and transport models.**

SUMMARY: Need Fluid-Based Modular Approach

- **Plasma kinetics is fundamental, but for time scales longer than species collision times (i.e., $t > 1/\nu_s$) 3-D ($\vec{x}$) fluid equations (extended MHD and transport) are feasible, appropriate, useful and needed. They are exact to extent that the relevant Chapman-Enskog kinetic equation (CEKE) can be solved for the kinetic distortion F_s which yields needed closure and collisional moments.**

- **Integrated simulations of tokamak plasmas need modular approach:**

  use small gyroradius expansion to order tokamak time scales, especially in the radial, parallel, and toroidal components of the plasma force balances,

  use extended MHD to check macrostability, and solve for order ρ_* flows plus evolving $\vec{B}$ field with reconnection regions and plasma responses to 3-D fields,

  solve relevant CEKE in flowing equilibrium and evolving $\vec{B}$ field to obtain collision-, fluctuation- and 3-D- induced closures and transport fluxes,

  solve tokamak plasma transport equations for n_e, $\Omega_{\rm tor}(E_\rho)$, p_e, p_i, $\psi_{\rm p}$,

  and iterate MHD, CEKE, closures and transport steps for self-consistency.